

Two hours

**THE UNIVERSITY OF MANCHESTER**

MATHEMATICS 0C1/1C1

20th January 2014

14.00 – 16.00

Answer SIX of the EIGHT questions

If more than SIX questions are attempted then credit  
will be given for the FIRST SIX answers.

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**Electronic calculators are not permitted**

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1. (1) Multiply out the brackets from the following expressions and collect terms.

(i)  $(x^2 - 4)(x + 3)$

(ii)  $(a - b - 2)(a + b - 1)$

(iii)  $(2 - x)(1 - (x - 1))$

(iv)  $(1 - x)(2x - 1)^2$

[4 marks]

(2) In 1(iv) above what is the term in  $x$ ? What is the coefficient of  $x^3$ ?  
What is the constant term?

[3 marks]

(3) Express each of the following in the form  $x^k$  where  $k$  is an integer or a rational number in its simplest form:

(i)  $\frac{x^2}{x^5}$       (ii)  $x^{-1}\sqrt[4]{x}$       (iii)  $(x^5)^{3/10}$

[3 marks]

2. Solve the following equations for  $x$ . (Find *all* solutions.)

(1)  $x^2 - 7x + 10 = 0$

(2)  $3x^2 + x - 2 = -x^2 + 2x - 1$

(3)  $\frac{x + 6}{x - 2} = \frac{x - 9}{3}$

(4)  $\frac{2}{x} - \frac{1}{x - 2} = \frac{1}{x - 6}$

(5)  $x^6 - 7x^3 - 8 = 0$

[2 marks for each part]

3. Solve the following equations for  $x$ . (Find *all* solutions.)

(1)  $16^x = 4$

(2)  $\log_2 \left( \frac{2}{x - 1} \right) = -3$

(3)  $\log_3 (9^{x+1}) = 3x$

(4)  $x \log_x (4) = \log_x (3)$

(5)  $\log_x (x - 1) + \log_x (3) = 1$

[2 marks for each part]

- 4.(1) Find the equation of the line  $C$  passing through the points  $(-2, 14)$  and  $(3, 4)$ .
- (2) Find the equation of the line  $D$  through the point  $(2, 1)$  which is parallel to the line  $C$ .
- (3) Find the equation of the line  $E$  through the point  $(2, 1)$  which is perpendicular to the line  $C$ .
- (4) At what point does the line  $E$  intersect the line  $C$ ?
- (5) What is the distance between the parallel lines  $C$  and  $D$ ?

[2 marks for each part]

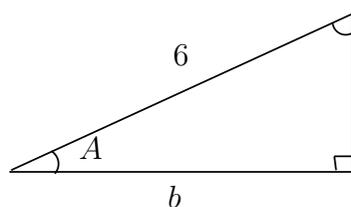
5. Let  $\mathcal{C}$  and  $\mathcal{D}$  be the curves given by  $y = -x^2 + 6x - 7$  and  $y = x^2 - 2x - 1$  respectively.

- (1) Find the two points where these curves cross. [2 marks]
- (2) Show that the point  $(-1, 2)$  is on the curve  $\mathcal{D}$  and find the equation of the tangent to  $\mathcal{D}$  at that point. [3 marks]
- (3) At what value of  $x$  do these two curves have the same slope? What value must the constant  $c$  be for the curve  $y = ce^x$  to also have this same slope at this same value of  $x$ ? [5 marks]

6. (1) Let  $f(x) = \frac{1}{1+x} - 1$ .

- (i) What is the domain of  $f$ ? [1 mark]
- (ii) What is  $f(f(x))$ ? [2 marks]
- (iii) Find  $f^{-1}(x)$ . [2 marks]

(2) The right angled triangle below has hypotenuse of length 6 and  $\cos(A) = 1/3$ .



Find:

- (i)  $b$       (ii)  $\sin(A)$       (iii)  $\tan(A)$       (iv)  $\sin(2A)$       (v)  $\cos(A - \pi/2)$

[1 mark for each part]

7. (1) Differentiate the following functions

(i)  $y = 5x^6 - 6$

(ii)  $y = x^{-1/4}$

(iii)  $y = e^{2x+1}$

[1 mark each]

(2) Find and classify the two stationary points of the function  $f(x) = 2x^3 + 3x^2 - 12x - 7$ .

[4 marks]

Sketch the graph of this function and using this graph indicate why the equation

$$2x^3 + 3x^2 - 12x - 7 = 0$$

has 3 solutions.

[3 marks]

8. Differentiate the following functions

(1)  $y = (2x + 1)^{-2}$

(2)  $y = \sin^2(x)$

(3)  $y = \frac{x}{1 - 2x}$

(4)  $y = \ln(2 + \cos(x))$

(5)  $y = \sqrt[3]{1 + e^x}$

[2 marks each]

**END OF EXAMINATION PAPER**