

Two hours

**THE UNIVERSITY OF MANCHESTER**

MATHEMATICS 0C1/1C1

18th January 2011

2.00 – 4.00

Answer **SIX** of the eight questions

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**The use of calculators is not permitted**

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1. (1) Multiply out the brackets from the following expressions and collect terms.

(i)  $(x - 3)(x^2 + 5)$

(ii)  $(a - b - 2)(a + b - 1)$

(iii)  $(1 - x)(2 - (x + 3))$

(iv)  $x(x - 1)(1 - 3x)$

[4 marks]

(2) In 1(iv) above what is the term in  $x^3$ ? What is the coefficient of  $x$ ?  
What is the constant term?

[3 marks]

(3) Express each of the following in the form  $x^k$ , where  $k$  is a rational number in its simplest form:

(i)  $\frac{x^4}{x^7}$       (ii)  $x^2\sqrt[3]{x}$       (iii)  $(x^4)^{1/6}$

[3 marks]

2. Solve the following equations for  $x$ . (Find *all* the solutions.)

(1)  $x^2 + 3x - 10 = 0$

(2)  $3x^2 - x - 5 = x^2 - 2x - 3$

(3)  $\frac{x + 1}{x - 3} = \frac{x - 1}{3}$

(4)  $\frac{1}{4 - x} + \frac{1}{x} = \frac{-2}{x + 8}$

(5)  $x^4 - 3x^2 + 2 = 0$

[2 marks for each part]

3. (1) Solve the following equations for  $x$ . (Find *all* solutions.)

(i)  $25^x = 5$

(ii)  $\log_5 \left( \frac{4}{x - 1} \right) = -1$

(iii)  $\log_3 (9^{x+2}) = 3x$

(iv)  $x \log_2 (x) = \log_3 (x)$

(v)  $\log_x (x^2 - 5x + 9) = 1$

4. (1) Find the equation of the line  $\mathcal{C}$  passing through the points  $(-1, 4)$  and  $(1, 8)$ . [2 marks]
- (2) Show that the point  $(-2, 2)$  lies on this line. [1 mark]
- (3) At what point  $A$  does the line  $\mathcal{C}$  cross the  $x$  axis? [2 marks]
- (4) What is the distance between the points  $A$  and  $(1, 8)$ ? [1 mark]
- (5) By considering the triangle formed from the points  $A$ ,  $(1, 8)$  and  $(1, 0)$  find the cosine of the angle between the line  $\mathcal{C}$  and the  $x$  axis. [2 marks]
- (6) Find the point of intersection of the line  $\mathcal{C}$  with the line  $y = 14 - 2x$ . [2 marks]
5. Let  $\mathcal{C}$  be the curve  $y = 3x + 1 - x^2$  and let  $\mathcal{E}$  be the line  $y = 5 - x$ .
- (1) Find the point where  $\mathcal{E}$  intersects  $\mathcal{C}$ . (Call this point  $A$ .) [2 marks]
- (2) Show that  $\mathcal{E}$  is the tangent to  $\mathcal{C}$  at  $A$ . [2 marks]
- (3) Find the equation of the normal to  $\mathcal{E}$  which passes through the point  $A$ . [2 marks]
- (4) Find the other point  $B$  at which this normal intersects  $\mathcal{C}$  [2 marks]
- (5) Find the equation of the tangent to  $\mathcal{C}$  at  $B$ . [2 marks]
6. (a) A right angled triangle has hypotenuse of length 7 and one angle of this triangle has cosine  $2/3$ . Find the sine of this angle, the lengths of the other two sides and the sine and cosine of the remaining angle. [5 marks]
- (b) State a formula relating  $\cos(A)\cos(B)$  to  $\cos(A+B)$  and  $\cos(A-B)$ . [1 marks]
- Using this formula and the fact that  $\cos(\pi/4) = 1/\sqrt{2}$  and  $\cos(0) = 1$  show that
- (i)  $\cos(\pi/8) = \sqrt{\left(\frac{1 + \sqrt{2}}{2\sqrt{2}}\right)}$  [2 marks]
- (ii)  $\cos(3\pi/8) = (\sqrt{2} - 1)\cos(\pi/8)$  [2 marks]

7. (1) Differentiate the following functions

(i)  $y = 3x^3 - 3$

(ii)  $y = \sqrt[3]{x}$

(iii)  $y = e^{2x-1}$

[1 mark each]

(2) Find and classify the two stationary points of the function

$$f(x) = 2x^3 - 3x^2 - 12x + 20. \quad [4 \text{ marks}]$$

Sketch the graph of this function and using this graph indicate why the equation

$$2x^3 - 3x^2 - 12x + 20 = -15$$

has only one solution.

[3 marks]

8. Differentiate the following functions

(1)  $y = (x + 1)^2 \sin(x)$

(2)  $y = \cos^2(x)$

(3)  $y = \frac{x + 1}{x + 2}$

(4)  $y = \ln(\sin(x))$

(5)  $y = \sqrt{e^x + 1}$

[2 marks each]

**END OF EXAMINATION PAPER**