Two hours

THE UNIVERSITY OF MANCHESTER

MATHEMATICS 0C1/1C1

18th January 2010

9.45 – 11.45

Answer **SIX** questions

The use of calculators is not permitted

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1. (1) Remove the brackets from the following expressions by multiplying out.

(i) $(x^2-1)(x+5)$

(ii) $(a-b-1)(a+b-2)$

(iii) $(1-a)(b-(a-1))$

(iv) $x(x+1)(1-3x)$

[4 marks]

(2) In 1(iv) above what is the term in x^2 ? What is the coefficient of x ? What is the constant term?

[3 marks]

(3) Express each of the following in the form x^k where k is a rational number:

(i) $\dfrac{x^3}{x^6}$ (ii) $x\sqrt[3]{x}$ (iii) $(x^6)^{1/3}$

[3 marks]

2. Solve the following equations for x. (Find *all* the solutions.)

(1) $x^2-6x+8=0$

(2) $4x^2+2x-3=2x^2-x-2$

(3) $\dfrac{x+2}{x-2}=\dfrac{x}{3}$

(4) $\dfrac{1}{3-x}+\dfrac{1}{x+1}=\dfrac{-2}{x+9}$

(5) $2^{2x}-2^{x+1}+1=0$

[2 marks for each part]

3. (1) Solve the following equations for x. (Find *all* solutions.)

(i) $8^x=2$

(ii) $\log_3\left(\dfrac{27}{x+1}\right)=2$

(iii) $\log_2\left(4^{x+1}\right)=x$

(iv) $\log_x\left(x^3-2x+1\right)=3$

[2 marks for each part]

(2) The volume $V(t)$ of porridge in a cooking pot at time t is related to the volume V_0 at time 0 by the equation $V(t)=V_0a^t$ for some $a>1$. If the volume doubles every 3 minutes what must a be? If the volume of the cooking pot is $5V_0$ how long is it before the pot overflows? [2 marks]

4. (1) Find the equation of the line $\mathcal{C}$ passing through the points $(-3,-3)$ and $(1,9)$. [2 marks]

(2) Show that the point $(-4,-6)$ lies on this line. [1 mark]

(3) At what point A does the line $\mathcal{C}$ cross the x axis? At what point B does the line $\mathcal{C}$ cross the y axis? [2 marks]

(4) What is the distance between the points A and B? [1 mark]

(5) By considering the triangle formed from the points A, B and $(0,0)$ find the sine of the angle between the line $\mathcal{C}$ and the x axis. [2 marks]

(6) Find the point of intersection of the line $\mathcal{C}$ with the line $y = 11x - 10$. [2 marks]

5. Consider the curves $\mathcal{D}$ and $\mathcal{E}$ given by $y = 2x^2 + x - 1$ and $y = x^2 + 2x + 1$ respectively.

(1) Find the two points where these curves cross. [3 marks]

(2) At what value of x do these two curves have the same slope? [2 marks]

(3) Show that the point $(-2,1)$ is on the curve $\mathcal{E}$ and find the equation of the tangent to $\mathcal{E}$ at that point [3 marks]

(4) Find the equation of the line through $(-2,7)$ which is normal to this tangent to $\mathcal{E}$ at $(-2,1)$. [2 marks]

6. (1) Differentiate the following functions

(i) $y = 4x^3 + 3$

(ii) $y = \sqrt[4]{x}$

(iii) $y = \cos(2x - 1)$ [1 mark each]

(2) Find and classify the two stationary points of the function

$$f(x) = x^3 + 3x^2 - 9x + 5. \quad \text{[4 marks]}$$

Sketch the graph of this function and using this graph indicate why the equation

$$x^3 + 3x^2 - 9x + 6 = 0$$

has only one solution. [3 marks]

7. Differentiate the following functions

(1) $y = (x-1)^2 e^x$

(2) $y = x(1 - \ln x)$

(3) $y = \dfrac{x-1}{x+1}$

(4) $y = \ln(\cos(x))$

(5) $y = e^{\sqrt{x}}$

[2 marks each]

8. (1) Find the indefinite integral

$$\int \left(x + \frac{1}{\sqrt{x}}\right)^2 dx.$$

[3 marks]

(2) Evaluate the definite integral

$$\int_0^{\pi/2} e^x + \cos(x)\, dx$$

[4 marks]

(3) Find the area bounded by the curve $y = x^2 + 1$, the y axis and the lines $x = -1$ and $x = 1$.

Using this answer, or otherwise, find the area bounded by the curve $y = x^2 + 1$ and the line $y = 2$. [3 marks]

END OF EXAMINATION PAPER