Feedback on the 2011 MATH43032/63032 exam

A1 Generally well done, though some students wasted time writing out the details in (i)-(iii), it was enough to simply say 'true' or 'false'.

A2 (a) Generally well done again. If I'd been answering this question I'd have set $\alpha_1 = p \wedge q$, $\alpha_2 = p \wedge \neg q$ etc. and then used these α's in what follows because it cuts down the subsequent notation and makes it easier to read. Those students who stuck with the $p \wedge q$ etc. though generally suffered no errors as a result.

Part (b) was tougher (that's why I indicated the number of marks for this part to avoid students spending inordinately much time on it). A few students got it though.

A3 This should have been easy, just argue semantics in (i) and (ii), and for students who could think and express themselves logically it was easy.

A4 As usual quite a few students couldn't say clearly what a *proof* (in this case in D) was, despite this question coming up very regularly! I was pleased that most students did produce the required formal proof. Many of those who didn't seemed at a lost as to how to set out such a proof, more practiced on the examples sheets beforehand would have helped them.

A5 The students were warned that I expected them to know the statement of McNaughton's Theorem, unfortunately many didn't heed the warning. This was a shame because quite a few marks in my exams can be gained simply by remembering key theorems, definitions etc. The last part followed in 2 lines from McNaughton's Theorem (since it tells us $w(\theta) = F_\theta(w(p)) = F_\theta(1/3) = n \times (1/3) + m \in [0, 1]$ for some $n, m \in \mathbb{N}$) but many students set of on a long time consuming proof by induction on $|\theta|$.

A6 Mostly C1-4 and the Mostert-Shields Theorem were given correctly but then applying the theorem wasn't so well done. Some students started trying to show that G satisfied C1-4 but this was already given (and in consequence $G(0, 0) = 0$, $G(1, 1) = 1$). What students should first have determined were the $x \in (0, 1)$ for which $G(x, x) = x$, which simplified to

$$x = \frac{1}{2}(x^2 + 2x - 1)$$

At this point several students managed get the arithmetic all wrong and find solutions in $(0, 1)$ when there were none. Amongst those who did correctly deduce that the only solutions to $G(x, x) = x$ were $0, 1$, so $\langle [0, 1], G, < \rangle$ is either isomorphic to $\langle [0, 1], \times, < \rangle$ or to $\langle [0, 1], \max\{x + y - 1, 0\}, < \rangle$ they then inexplicably made the wrong choice despite having earlier correctly stated in the Mostert-Shileds Theorem how it can be determined (since, for example, $G(1/3, 1/3) = 0$, it must be the second one).

B7 The first part was not that well done in general, despite all the practice the students had had with this sort of question. A common mistake was to omit the case when $s_i \cap S_\eta = s_i \cap At^L = \emptyset$ (with η a tautology) for all i. In the other case a key point to notice was that when $s_i \cap At^L \neq \emptyset$ for some least i then $s_j = \emptyset$ for $j < i$ so *if* $s_i \cap S_{\neg\phi} \neq \emptyset$ then i must also the the least such that this holds (and so $\emptyset \neq s_i \cap S_{\neg\phi} \subseteq S_{\neg\theta}$ etc.). Some students failed to notice this point.

The second part was generally well done. Pretty much all the students knew what to do: try $\theta = p$, $\phi = q$, $\psi = r$ and write down $s_1, s_2, \ldots$, but surprisingly many, to their cost, didn't do a quick mental check that their $s_1, s_2, \ldots$ actually did provide a counter-example!

B8 The students who tried this question generally did well on the first and last parts. The middle part was more challenging and most left it out though those who had the confidence to try it were largely successful.

B9 Parts (a) and (b) were generally well done. My advice in demonstrations such as part (a) required would be to split it into sensible cases (like $w(q) \leq w(p)$ and $w(p) < w(q)$). Students who didn't do this tended to land up with some slightly messy arithmetic though in this case they usually managed to avoid errors (unlike in A6).

The last part was short but tricky (and to avoid unfairness only worth 3 marks, unlike the 4 available for the other bits). I was pleased that several students managed it with aplomb!